To **solve** an equation, we need to get the $x$ by itself – when it’s by itself, we get the answer of what $x$ is! In order to get the $x$ by itself, we need to **rearrange** the numbers and symbols in the equation while still keeping the equation accurate.

Think of the equals sign in the equation as a balance scale. We can change the positions of items on the scales, and take items on or off – we can change the position of numbers and symbols in the equation, and remove numbers or add them on – as long as we keep the scales balanced. We keep the scales balanced by **always doing the same thing to both sides of the equation**.

![Balance scales](image)

Remember also that the sign of a variable or constant is what is in **front** of it; sometimes it’s an “invisible +” if it’s at the beginning.

To **solve** an equation, **rearrange** so that all variable parts (anything with $x$ in) are on one side of the equal sign, and all number parts (parts with just numbers, not $x$’s) are on the other side. To do this rearranging, you need to identify what operations are being used (Add, Subtract, Multiply, Divide) and “Undo” operations by using opposite operations. **Remember: Whatever you do to one side, you must do to the other side to keep equation balanced.**

**Example 1: Solve** $4x - 5 = 15$

<table>
<thead>
<tr>
<th>Step</th>
<th>Equation</th>
<th>Action</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial</td>
<td>$4x - 5 = 15$</td>
<td></td>
</tr>
<tr>
<td>+ 5</td>
<td>$4x = 20$</td>
<td>Add 5 to both sides because this will remove the minus 5 from the LHS and leave just the $4x$ (add and subtract are opposite operations).</td>
</tr>
<tr>
<td>+ 5</td>
<td>$4x = 20$</td>
<td>Simplify – notice there’s now only $x$ parts on LHS and number parts on RHS.</td>
</tr>
<tr>
<td>$\frac{4x}{4}$</td>
<td>$\frac{20}{4}$</td>
<td>Divide both sides by 4 because 4 is multiplied by $x$, so the opposite operation – division by 4 – will remove the 4 and leave only $x$.</td>
</tr>
<tr>
<td>$x$</td>
<td>$5$</td>
<td>Simplify – notice we now have the $x$ by itself and our <strong>answer is</strong> $x = 5$.</td>
</tr>
</tbody>
</table>

Let’s check our answer in the original problem by replacing $x$ with 5: $4 \times 5 - 5 = 20 - 5 = 15$. 

Our answer makes LHS = RHS in this equation so our answer is correct.
The most important rule to remember is to **do the same thing to both sides of the equation**. This preserves equality.

### Example 2: Solve \( \frac{x}{3} + 4 = 9 \)

1. **Step 1:** Subtract 4 from both sides because add and subtract are opposite operations, so subtracting 4 removes plus 4 from LHS and leaves just \( \frac{x}{3} \).

   \[
   \frac{x}{3} + 4 - 4 = 9 - 4
   \]

2. **Step 2:** Simplify – notice there’s now only \( x \) parts on LHS and number parts on RHS.

   \[
   \frac{x}{3} = 5
   \]

3. **Step 3:** Multiply by 3 on both sides because \( x \) is divided by 3, so the opposite operation – multiplication by 3 – will remove the 3 and leave only \( x \).

   \[
   \frac{x}{3} \times 3 = 5 \times 3
   \]

4. **Step 4:** Simplify - notice we now have the \( x \) by itself and our answer is \( x = 15 \).

   \[
   x = 15
   \]

Let’s check our answer in the original problem by replacing \( x \) with 15: \( \frac{15}{3} + 4 = 5 + 4 = 9 \).

### Example 3: Solve \( \frac{5 + 3x}{2} + 5 = 3x \)

1. **Step 1:** Subtract 5 from both sides because add and subtract are opposite operations. So subtracting 5 removes plus 5 from LHS and leaves just \( \frac{5 + 3x}{2} \).

   \[
   \frac{5 + 3x}{2} + 5 - 5 = 3x - 5
   \]

2. **Step 2:** Multiply by 2 on both sides because \( 5 + 3x \) is divided by 2, so the opposite operation – multiplication by 2 – will remove the 2 and leave only \( x \). *NOTE: \( \frac{2}{1} \) is the same as 2, since 2 divided by 1 equals 2.*

   \[
   \left( \frac{5 + 3x}{2} \right) \times 2 = (3x - 5) \times \frac{2}{1}
   \]

3. **Step 3:** Simplify by multiplying LHS and expanding brackets RHS. Then subtract 3\( x \) from both sides since add and subtract are opposite operations, so subtracting 3\( x \) removes +3\( x \) from LHS.

   \[
   5 + 3x - 3x = 6x - 5 - 10
   \]

4. **Step 4:** Add 10 to both sides, removing -10 from RHS and rearranging equation with only \( x \) parts on LHS and number parts on RHS.

   \[
   5 + 10 = 3x + 10
   \]

5. **Step 5:** Divide both sides by 3 because 3 is multiplied by \( x \), so the opposite operation will remove the 3 and leave only \( x \).

   \[
   \frac{5}{3} = \frac{3x}{3}
   \]

6. **Step 6:** Our answer is: \( x = 5 \)

   \[
   x = 5
   \]

Let’s check our answer in the original problem by replacing \( x \) with 5: \( \frac{5 + 3 \times 5}{2} + 5 = \frac{20}{2} + 5 = 15 \), RHS: \( 3 \times 5 = 15 \)

Note both sides equal so answer is right.
Example 4: Solve $10y - (4y + 8) = -20$

$10y - (4y + 8) = -20$

Distribute -1 on the left side.

$10y + (-1)(4y) + (-1)(8) = -20$

Simplify.

$10y - 4y - 8 = -20$

Add 8 to both sides to get $6y$ by itself.

$6y - 8 = -20 + 8$

$6y = -12$

Divide both sides by 6 to get $y$ by itself.

$y = -2$

ANSWER

Let’s check our answer in the original problem by replacing $y$ with -2:

LHS: $10 \times (-2) - (4 \times (-2) + 8) = -20 - (8 + 8) = -20 - 0 = -20$

Making a variable the subject of an equation

Sometimes a question asks you to make a variable the subject of an equation. This means you need to get a variable by itself on one side of the equals sign, so it’s just like solving an equation. For example, if $Q = 110 - 4P$, and you are asked to make $P$ the subject of the equation, the way to do this is just to solve the equation – i.e. to get $P$ by itself on one side of the equals sign.

Example 5: Make $P$ the subject of $Q = 110 - 4P$

$Q = 110 - 4P$

Subtract 110 from both sides to get $4P$ by itself.

$Q - 110 = 4P$

Divide both side by 4 to get $P$ by itself.

$\frac{Q - 110}{4} = P$

OR

$P = \frac{Q - 110}{4}$

ANSWER
Practice Questions

Solve:
1. \(2x - 5 = 17\)
2. \(3y + 7 = 25\)
3. \(5n - 2 = 38\)
4. Rearrange this formula \(A = 2a^2 + 4ab\) so that \(b\) is the subject of the formula.
5. \(s = ut + \frac{1}{2}at^2\) is a formula used in Physics to calculate distance. Make "\(a\)" the subject of the formula.

\[
\frac{\sqrt{\frac{n}{s}}}{\sqrt{\frac{n}{s}}} = a
\]

And we get:
\[
\frac{\sqrt{n}}{\sqrt{s}} = a
\]
Divide both sides by \(\sqrt{z}\)
\[
\frac{\sqrt{n}}{\sqrt{s}} = a
\]
Divide both sides by \(\sqrt{z}s\)
\[
\frac{\sqrt{n}}{\sqrt{s}} = a
\]
Multiply both sides by \(\sqrt{z}s\)
\[
\frac{\sqrt{n}}{\sqrt{s}} = a
\]
Swap sides
\[
\frac{\sqrt{n}}{\sqrt{s}} = a
\]
Swap sides
\[
\sqrt{n} = \sqrt{s}a
\]
Subtract \(\sqrt{s}a\) from both sides
\[
\sqrt{n} - \sqrt{s}a = 0
\]
Subtract \(\sqrt{s}a\) from both sides
\[
\sqrt{n} - \sqrt{s}a = 0
\]
Subtract \(\sqrt{s}a\) from both sides
\[
\sqrt{n} - \sqrt{s}a = 0
\]
Subtract \(\sqrt{s}a\) from both sides
\[
\sqrt{n} - \sqrt{s}a = 0
\]
Subtract \(\sqrt{s}a\) from both sides
\[
\sqrt{n} - \sqrt{s}a = 0
\]
Subtract \(\sqrt{s}a\) from both sides

Question 1:
\(x = 11\)

Question 2:
\(y = 6\)

Question 3:
\(a = 4b\)

Question 4:
\(\frac{3}{2}a^2 = 2a\)

Question 5:
\(x = \frac{1}{1}\)